

# CRITICAL BEHAVIOR OF LAYERED SUPERCONDUCTING FILMS IN PARALLEL MAGNETIC FIELD

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## Abstract

The equilibrium magnetization for layered superconducting films that experience a nonzero component,  $H_{\parallel}$ , of magnetic field applied parallel to the layers is computed at temperatures and at perpendicular field components in the vicinity of the decoupling transition. A fermion analogy is exploited for this purpose, whereby it is found that the parallel magnetization shows an anomalous  $H_{\parallel}^{-1}$  tail at high fields due to entropic fluctuations of the (parallel) lattice of Josephson vortices. A collective pinning effect is also identified for  $c$ -axis transport limited by a single planar defect oriented parallel to both  $c$  and to the applied magnetic field.

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## I. Introduction

It is well known that the electronic conduction in the normal state of high-temperature superconductors (HTSC) is confined primarily to the copper-oxygen planes common to these materials.<sup>1</sup> This suggests the Lawrence-Doniach (LD) model of a stack of Josephson-coupled superconducting layers below  $T_c$ .<sup>2,3</sup> The following question can then be posed:<sup>4</sup> Is the superconducting phase strictly three dimensional (3D), or does two-dimensional (2D) superconductivity appear somewhere in the phase diagram? The experimental answer is complex. Conductivity measurements give evidence for a 2D superconducting transition,<sup>5</sup> whereas thermodynamic measurements are more consistent with a 3D transition.<sup>6</sup> The resolution to this paradox is that, although the thermodynamic transition into the superconducting phase is in fact 3D,<sup>7–10</sup> quasi-2D behavior arises for physics on length scales small in comparison to the Josephson penetration length,<sup>11</sup>  $\lambda_J$ . Indeed, the  $c$ -axis Josephson plasma resonance seen in HTSC for magnetic fields oriented perpendicular to the layers that exceed the 2D-3D cross-over scale,<sup>12</sup>  $B_*^\perp \sim \Phi_0/\lambda_J^2$ , is an experimental example of such behavior,<sup>13</sup> in which case Josephson coupling persists in between layers while  $c$ -axis vortex lines have degenerated into decoupled pancake vortices.<sup>3</sup>

In this paper, we shall obtain the critical properties theoretically expected for thin films of extreme type-II layered superconductors in magnetic field aligned parallel to the layers. The quasi-2D regime cited above is presumed throughout. This guarantees that the thermodynamics “factorizes” into that associated with (perpendicular) pancake vortices and to that associated with (parallel) Josephson vortices.<sup>9,14</sup> In addition, we shall assume that the thickness of the film (including leads) is less than the London penetration length to insure the absence of magnetic screening across the layers. The theoretical analysis exploits a new fermion analogy for the LD model.<sup>15</sup> We first compute the parallel equilibrium magnetization. It is found to exhibit a series of renormalized melting transitions connected with the parallel vortex lattice,<sup>16,17</sup> as well as an anomalous  $H_\parallel^{-1}$  tail at high parallel fields,<sup>15</sup>  $H_\parallel$  (see Figs. 1 and 2). We then compute  $c$ -axis transport limited by the presence of a single planar defect or pair of edges oriented parallel to both the  $c$ -axis and to the applied magnetic field. A collective pinning regime is identified in the high-field limit (see Fig. 1) for planar pins with an effective thickness  $\xi_p > a_0$ , where  $a_0$  denotes the lattice constant of the parallel vortex lattice. In particular, the current-voltage ( $I$ - $V$ )

characteristic is found to be *algebraic* in this case, and to be inversely related to that obtained for a Luttinger liquid in the presence of a single backscattering impurity.<sup>18</sup>

## II. Parallel Equilibrium Magnetization

The problem to be solved then is the *parallel* thermodynamics of a finite number,  $N$  of Josephson coupled planes in the quasi-2D regime<sup>11–14</sup>  $H_\perp > \Phi_0/\lambda_J^2$  and/or  $T > T_{cr}$ , where the 2D-3D cross-over temperature  $T_{cr}$  marks the point at which the intra-plane phase coherence length,  $\xi$ , matches the Josephson penetration length,  $\lambda_J$ . It is implicit then that long-range in-plane phase coherence is lost at a temperature  $T_c$  below  $T_*$ , at which point the planes entirely decouple;<sup>7–9</sup> i.e.,  $\xi(T) = \infty$  for  $T < T_c$ , while  $\lambda_J(T) = \infty$  for  $T > T_*$ . Since both  $\xi$  and  $\lambda_J$  are finite in the interval between  $T_c$  and  $T_*$ , the relationship  $\xi(T_{cr}) \sim \lambda_J(T_{cr})$  then implies the inequalities  $T_c < T_{cr} < T_*$ .<sup>11</sup> We shall now introduce the corresponding LD free-energy functional, which reads

$$E_{LD} = J_\parallel \int d^2r \left[ \sum_{l=1}^N \frac{1}{2} (\vec{\nabla} \theta_l)^2 - \Lambda_0^{-2} \sum_{l=1}^{N-1} \cos(\theta_{l+1} - \theta_l - A_z) \right] \quad (1)$$

in the absence of magnetic screening. Here,  $\theta_l(\vec{r})$  denotes the phase of the superconducting order parameter in layer  $l$ , where  $\vec{r} = (x, y)$  is the planar coordinate. The parallel magnetic induction  $B_\parallel = (\Phi_0/2\pi d)b_\parallel$  found between layers  $l$  and  $l+1$  and aligned along the  $y$  axis is related to the vector potential above by  $A_z = -b_\parallel x$ . Here  $d$  represents the spacing between layers. Last,  $J_\parallel$  is a measure of the local in-plane phase rigidity, while  $\Lambda_0$  sets the bare scale for the Josephson penetration length. The author has recently obtained as analogy between the above LD model and coupled chains of spinless fermions at zero temperature, where each chain corresponds to a layer.<sup>15</sup> Specifically, the Hamiltonian for the fermion model is divided into two parts,  $H = H_\parallel + H_\perp$ , with

$$H_\parallel = \sum_{l=1}^N \int dx \left[ v_F \left( \Psi_L^\dagger i \partial_x \Psi_L - \Psi_R^\dagger i \partial_x \Psi_R \right) + U_\parallel \Psi_L^\dagger \Psi_R^\dagger \Psi_L \Psi_R \right] \quad (2a)$$

and

$$H_\perp = U_\perp \sum_{l=1}^{N-1} \int dx \left[ \Psi_L^\dagger(x, l) \Psi_R^\dagger(x, l+1) \Psi_L(x, l+1) \Psi_R(x, l) + \text{H.c.} \right], \quad (2b)$$

and with field operators  $\Psi_R(x, l)$  and  $\Psi_L(x, l)$  for right ( $R$ ) and left ( $L$ ) moving fermions. The coordinate along the Josephson vortices,  $y$ , is related to the imaginary time variable  $\tau$  of the fermion analogy by  $y = v'_F \tau$ . Here, the Fermi velocity  $v'_F = v_F \operatorname{sech} 2\phi$  is renormalized by the intra-chain interaction  $U_{\parallel}$ ,<sup>19</sup> with  $\tanh 2\phi = U_{\parallel}/2\pi v_F$ . Also,  $U_{\perp} > 0$  is a repulsive backscattering interaction energy<sup>19</sup> in between chains. The Gibbs free-energy of the LD model (1) with respect to the normal state is then found to be related to the ground-state energy  $E_F$  of the fermion analogy by

$$(G_s - G_n)/k_B T = (L_y/v'_F)[E_F(U_{\perp}) - E_F(0)]. \quad (3)$$

The identifications

$$b_{\parallel} = 2\pi(N_{l+1} - N_l)/L_x, \quad (4)$$

$$T = e^{2\phi} T_{*0}, \quad (5)$$

$$\Lambda_0^{-2} = \alpha^{-2} (2|U_{\perp}|/\pi v'_F)(T/T_{*0}), \quad (6)$$

complete the equivalence between the models, where  $N_l/L_x$  gives the fermion density in the  $l^{\text{th}}$  chain,  $T_{*0} = 4\pi J_{\parallel}$  is the decoupling temperature that marks the point at which interlayer phase coherence is lost,<sup>7-9</sup> and where  $\omega_0 = v'_F \alpha^{-1}$  is the ultraviolet cutoff in energy. This analogy is a direct generalization of the well-known equivalence that exists between the sine-Gordon model and the massive Thirring/Luther-Emery model in  $1 + 1$  dimensions to a layered structure.<sup>21</sup> It reduces to a free theory in the double-layer case ( $N = 2$ ) along the Luther-Emery (LE) line  $T = 2\pi J_{\parallel}$ .<sup>19</sup>

The general case, however, can be treated in the mean-field approximation defined by the charge-density wave (CDW) order parameter  $\chi_l(x) = \langle \Psi_R^{\dagger}(x, l) \Psi_L(x, l) \rangle$ , and the associated gap equation<sup>22</sup>

$$\Delta_l = U_{\parallel} \chi_l + U_{\perp} (\chi_{l+1} + \chi_{l-1}). \quad (7)$$

[The order parameters at the boundaries are set to  $\chi_0(x) = 0 = \chi_{N+1}(x)$ .] The mean-field Hamiltonian then has the form  $H_{\text{MF}} = \sum_{l=1}^N \int dx \Psi_l^{\dagger} (H_l - \mu_l) \Psi_l$ , where the spinor field,  $\Psi_l(x) = (\Psi_L(x, l), \Psi_R(x, l))$ , for each layer is acted upon by the one-body operator

$$H_l = \sigma_3 v_F i \partial_x + \sigma_+ \Delta_l(x) + \sigma_- \Delta_l^*(x). \quad (8)$$

Here we define  $\sigma_{\pm} = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$ , where  $\sigma_i$  represent the Pauli matrices. Also,  $\mu_l$  denotes the chemical potential for the fermions in layer  $l$ . At zero parallel magnetic induction, the mean-field ground state has spin-density wave (SDW) type order,  $\chi_l(x) = (-1)^l \chi_0$ , for  $U_{\perp} > 0$ . In the minimal case,  $N = 2$ , a constant gap  $\Delta_0 = \omega_0 / \sinh[2\pi v_F / (U_{\perp} - U_{\parallel})]$  opens at each Fermi surface for interaction parameters satisfying  $U_{\perp} > U_{\parallel}$ . The latter phase boundary agrees with renormalization group results.<sup>23</sup> Also, by Eqs. (3) and (4), the line-tension of a single Josephson vortex is given in general by  $\varepsilon_{\parallel} = k_B T |\Delta_{\sigma}| / v'_F$ , with a pseudo-spin gap equal to  $\Delta_{\sigma} = \Delta_0(U_{\perp}) - \Delta_0(0)$ . For weak coupling,  $U_{\perp} \rightarrow 0$ , we then have  $\Delta_{\sigma} = U_{\perp} \frac{\partial}{\partial U_{\perp}} \Delta_0|_{U_{\perp}=0}$  at temperatures  $T < T_{*0}$  (or  $U_{\parallel} < 0$ ), while  $\Delta_{\sigma} = \Delta_0$  at temperatures  $T > T_{*0}$  (or  $U_{\parallel} > 0$ ). The former yields a pseudo-spin gap of  $\Delta_{\sigma} \cong 0.9(U_{\perp}/2\pi\alpha)$  along the LE line  $T = 2\pi J_{\parallel}$  for the case  $N = 2$ , which is comparable to the exact value of  $\Delta_{\sigma} = U_{\perp}/2\pi\alpha$ .<sup>19</sup> The success that this mean-field approximation has in the double-layer case indicates that it is reliable for the case of large  $N$ , where fluctuations are smaller. A constant gap,  $\Delta_0 = \omega_0 / \sinh[2\pi v_F / (2U_{\perp} - U_{\parallel})]$ , opens at each Fermi surface in such case for interaction parameters satisfying  $U_{\perp} > \frac{1}{2}U_{\parallel}$ . The identification (6) then implies the leading dependence  $T_* = 4\pi J_{\parallel}[1 + \frac{1}{2}(\alpha/\Lambda_0)^2]$  of the decoupling temperature with the bare Josephson scale. Also, the pseudo-spin gap now has value  $\Delta_{\sigma} \cong 1.9(U_{\perp}/2\pi\alpha)$  along the LE line. The parallel lower critical field,  $H_{c1}^{\parallel} = 4\pi\varepsilon_{\parallel}/\Phi_0$ , vanishes exponentially at the decoupling temperature  $T_*$ , however, since  $\Delta_{\sigma} \propto \Delta_0(T)$ . This implies an inflection point in its temperature profile below  $T_*$ .<sup>14,24</sup>

The presence of a parallel magnetic field, however, will generally induce the phase  $\theta'_l(x)$  of the CDW order parameter,  $\chi_l = (-1)^l \chi_0 \exp(i\theta'_l)$ , to wind. This can be seen explicitly from the Ginzburg-Landau (GL) equations that describe the present mean-field theory at “temperatures”  $T' = v'_F/L_y$  near  $T'_c \sim \Delta_0$  [see Appendix A, Eq. (A10)]. After extending Gorkov’s original derivation<sup>25</sup> of the GL equations to the case of an isolated pair  $(l, l+1)$  of consecutive layers, one recovers the original LD free energy functional (1), but with the bare scale  $\Lambda_0$  replaced by the renormalized Josephson penetration length  $\lambda_J = v_F/|\Delta_{\sigma}|$ .<sup>14</sup> This is a result of entropic wandering of the Josephson vortices in the parallel direction, which is particularly relevant in the regime of low parallel fields,  $b_{\parallel} \ll \Lambda_0^{-1}$ . Note that the latter corresponds precisely to the long wave-length limit in which Gorkov’s derivation of the GL equations is valid.<sup>25</sup> Also, the CDW phase is related to the true phase in the LD model

(1) by the gauge transformation  $\theta_l = \theta'_l - 2k_{F,l}x$ , where  $k_{F,l}$  denotes the Fermi wave-vector of layer  $l$ . The formula  $N_l = \pi^{-1}k_{F,l}L_x$  then indicates that the number of fermions,  $N_l$ , is equal to the winding number of the CDW phase in a given chain  $l$ . This fact coupled with the identification (4) also demonstrates that the gauge-invariant phase difference between these layers is just  $\theta_{l+1} - \theta_l - A_z = \theta'_{l+1} - \theta'_l$ . A GL theory analysis<sup>16</sup> of the *renormalized* LD model<sup>14</sup> (1) then yields a sequence of first-order commensuration transitions at parallel fields  $H_{l_1}^{\parallel}$  between a parallel vortex lattice with flux penetration every  $l_1$  layers to one with flux penetration every  $l_1 + 1$  layers as the field is lowered. Fig. 1 displays the predicted phase diagram in the critical regime using the estimates  $H_1^{\parallel} = \frac{1}{3}\Phi_0/\lambda_J d$  and  $H_2^{\parallel} = \frac{3}{8}H_1^{\parallel}$  based on the continuum limit. Contrary to claims in the literature,<sup>8</sup> we therefore find no evidence for a true layer decoupling transition as a function of parallel field in the critical regime.<sup>14,15</sup>

Consider now a parallel vortex lattice in the limit of high parallel field, but where flux penetrates only every  $l_1 \geq 2$  layers. The LD model (1) indicates that spatial variations of the superconducting phase are generally absent in the former limit;<sup>14,16</sup> i.e.,  $\theta_l(x) = 0$  and  $\theta'_l(x) = 2k_{F,l}x$ . If there exists *no* flux penetration in between layers  $l$  and  $l \mp 1$ , the gap function (7) then has the form

$$\Delta_l = (-1)^{l \pm 1} U_{\perp} \chi_0 e^{i(\theta'_{l \pm 1} - \theta'_l)} = (-1)^{l+1} \Delta_{\sigma} e^{\pm i b_{\parallel} x} \quad (9)$$

along the special line  $U_{\perp} = U_{\parallel}$  in parameter space, where  $b_{\parallel}$  denotes the flux in between layers  $l$  and  $l \pm 1$  and where  $\Delta_{\sigma} = U_{\perp} \chi_0$ . Here, we have made a gauge transformation in order to set  $\mu_l$  and  $k_{F,l}$  to zero (see ref. 22). The mean-field Hamiltonian (8) then has energy eigenvalues

$$\varepsilon_k^{\pm} = v_F k_F \pm [v_F^2 (k - k_F)^2 + \Delta_{\sigma}^2]^{1/2}, \quad (10)$$

with an effective Fermi wave number  $k_F = \pm b_{\parallel}/2$  and a pseudo-spin gap  $\Delta_{\sigma}$ . The equilibrium magnetization  $M_{\parallel} = -\frac{\partial}{\partial H_{\parallel}}[(G_s - G_n)/V]$  can then be computed using the equivalence (3). After following steps similar to those taken in the double-layer ( $l, l \pm 1$ ) case along the LE line,<sup>15</sup> one obtains the formula

$$-4\pi M_{\parallel}^{(l_1)} = \frac{1}{2} H_{c1}^{\parallel} \left\{ \left[ 1 + \left( \frac{H_{\parallel}}{l_1^{-1} B_*^{\parallel}} \right)^2 \right]^{1/2} - \frac{H_{\parallel}}{l_1^{-1} B_*^{\parallel}} \right\} \quad (11)$$

for the equilibrium parallel magnetization, where  $B_*^\parallel = \Phi_0/\pi\lambda_J d$  is the parallel field cross-over scale, with Josephson penetration length  $\lambda_J = v_F/|\Delta_\sigma|$ . Given the phase diagram (Fig. 1) arrived at by the previous low-field analysis, this result implies a series of jumps *down* in  $M_\parallel$  at each melting field,  $H_{l_1}^\parallel$ , as parallel field increases (see Fig. 2). Notice that the sign of the jump in this case is opposite to that observed in HTSC for vortex lattice melting as the perpendicular magnetic field is swept.<sup>26</sup> Last, (9) indicates that the gap function  $\Delta_l(x)$  simply acquires a constant phase factor upon a uniform translation  $x \rightarrow x + a_l$  of the coordinate in layer  $l$ . Hence, the parallel vortex lattice is infinitely smectic in the high-field limit. (This result is consistent with the shear instability obtained in the standard GL analysis of LD model (1).<sup>3</sup>) The parallel magnetization in the high-field limit ( $l_1 = 1$ ) is then given by that of an isolated double layer,<sup>15</sup> which coincides with (11) up to prefactor of  $2^{1/2}$ . This means that the parallel magnetization must show an anomalous  $H_\parallel^{-1}$  tail at high fields.

Finally, to demonstrate that the above formula (11) for the parallel magnetization in the high-field limit is generic, we shall now analyze the spectrum of the mean-field Hamiltonian (8) along another special line,  $U_\parallel = 0$ , in which case degenerate perturbation theory in powers of  $U_\perp$  can be employed. Let us suppose again that flux penetrates only every  $l_1 \geq 2$  layers, and that there exists *no* flux penetration in between layers  $l$  and  $l \mp 1$ . After performing the same gauge transformation as before to set  $k_{F,l}$  and  $\mu_l$  to zero (see ref. 22), we obtain a gap function (7) of the form  $\Delta_l = (-1)^{l+1} \Delta_\sigma (1 + e^{\pm i b_\parallel x})$ , where  $b_\parallel$  denotes the flux in between layers  $l$  and  $l \pm 1$ , and where  $\Delta_\sigma = U_\perp \chi_0$ . Now in the absence of Josephson coupling,  $U_\perp = 0$ , the mean-field Hamiltonian (8) has eigenstates  $\Psi_a = (0, e^{i k x})$  and  $\Psi_b = (e^{i(k-2k_F)x}, 0)$ , with the corresponding energy eigenvalues  $\varepsilon_a = v_F k$  and  $\varepsilon_b = v_F(2k_F - k)$ . As before, we have  $k_F = \pm b_\parallel/2$ . The application of degenerate perturbation theory in powers of  $U_\perp$  with respect to such states at momenta  $k \sim k_F$  then yields the well-known formula

$$\varepsilon_k^\pm = \frac{1}{2}(\varepsilon_a + \varepsilon_b) \pm \left[ \frac{1}{4}(\varepsilon_a - \varepsilon_b)^2 + \Delta_\sigma^2 \right]^{1/2}$$

for the perturbed energy eigenvalues. Substitution of the unperturbed energies above then yields the previous result (10). We thereby recover the formula (11) for the parallel magnetization in the case that  $l_1 \geq 2$ . Last, when the parallel flux penetrates all layers ( $l_1 = 1$ ), the gap function (7) has the form  $\Delta_l = (-1)^{l+1} \Delta_\sigma (e^{\mp i b_\parallel x} + e^{\pm i b_\parallel x})$ . This means

that we must take into account the previous unperturbed states, as *well* as their time-reversed counterparts obtained after making the (global) replacement  $k_F \rightarrow -k_F$ . The end result is again the previous formula (11) for the parallel magnetization in the case  $l_1 = 1$ .

### III. Parallel Collective Pinning

We shall now compute  $c$ -axis transport limited by a single planar defect or pair of edges in the high-field limit,  $H_{\parallel} > H_1^{\parallel}$ , where parallel flux penetrates in between every layer. The planar pin is assumed to be parallel to both the  $c$ -axis and to the applied magnetic field. Due to the extreme smecticity symptomatic of this regime, it is sufficient to study the double-layer case. The LD model (1) then reduces to a pinned sine-Gordon system<sup>14</sup> with free-energy functional

$$E_{\text{SG}} = J_{\parallel} \int d^2r \left[ \frac{1}{4} (\vec{\nabla} \theta_- - \hat{x} b_{\parallel})^2 - \Lambda_0^{-2} \cos \theta_- \right] - \varepsilon_p^{\parallel} \int dy \cos \theta_-|_{x_0} + \varepsilon_L^{\parallel} \int d^2r \theta(x - x_0) \frac{\partial \theta_-}{\partial x}, \quad (12)$$

where  $\theta_-$  is the gauge-invariant phase difference in between the consecutive layers. Here,  $\varepsilon_L^{\parallel} = c^{-1}(I/L_y)(\Phi_0/2\pi)$  is the line tension due to the Lorentz force in the absence of flux flow. The Lorentz force and the pinning force are in equilibrium in such case, which implies that the  $c$ -axis current  $I$  flows *only* at the pin site  $x_0$ .<sup>27</sup> Also, since Josephson coupling is weaker near the pin, we have that  $\varepsilon_p^{\parallel} < 0$ . Following Coleman<sup>21</sup> and LE,<sup>19</sup> this model is equivalent to the massive Thirring model for 1D fermion fields,  $\Psi = (\Psi_L, \Psi_R)$ , in the presence of a single backscattering impurity.<sup>18</sup> Its Hamiltonian description is then

$$H_{\sigma} = \int dx \Psi^{\dagger} (\sigma_3 v'_F i \partial_x + \sigma_1 \Delta_{\sigma} + 2^{1/2} V_{\text{KF}}) \Psi + 2g_0 \int dx \Psi_L^{\dagger} \Psi_R^{\dagger} \Psi_R \Psi_L - \xi_p \Delta_{\sigma} (\Psi_L^{\dagger} \Psi_R + \Psi_R^{\dagger} \Psi_L)|_{x_0}, \quad (13)$$

where  $V_{\text{KF}}(x) = 2\pi v'_F (\varepsilon_L^{\parallel}/k_B T) \theta(x - x_0)$  is the voltage drop equivalent to the Lorentz force located at  $x_0$ , while  $\xi_p \propto -\varepsilon_p^{\parallel}$  gives the effective thickness of the pin plane. The interaction between fermions is related to the depinning temperature (or LE line)  $T_{dp} = 2\pi J_{\parallel}$  by the relationship

$$\frac{T_{dp}}{T} = 1 + \frac{g_0}{\pi v'_F}. \quad (14)$$

Notice then that the fermions interact repulsively for  $T < T_{dp}$ , while they interact attractively for  $T > T_{dp}$ . Last, Eq. (4) indicates that the total number of fermions is equal to the total number of Josephson vortices lying in between the consecutive layers.

Yet Eq. (14) indicates that the Thirring model is nearly free at temperatures near  $T_{dp}$ , with a Fermi surface at  $k_F = \frac{1}{2}b_{\parallel}$ . This suggests a canonical transformation of the form

$$c_k = x_k a_k + y_k b_k \quad (15)$$

$$d_k = -y_k a_k + x_k b_k \quad (16)$$

to remove the (mass) gap term in (13) at momenta  $k$  in the vicinity of the Fermi surface. Here, the original field operators for right and left moving spinless fermions are represented as  $\Psi_R(x) = L_x^{-1/2} \sum_k e^{ikx} a_k$  and  $\Psi_L(x) = L_x^{-1/2} \sum_k e^{ikx} b_k$ , respectively. The former is achieved by the choice of coherence factors  $(x_k, y_k) = (u_k, v_k)$  for  $k > 0$  and  $(x_k, y_k) = (v_k, -u_k)$  for  $k < 0$ , with

$$u_k = 2^{-1/2} \left( 1 + \frac{v'_F k}{E_k} \right)^{1/2}, \quad (17)$$

$$v_k = 2^{-1/2} \left( 1 - \frac{v'_F k}{E_k} \right)^{1/2}, \quad (18)$$

along with energy eigenvalue

$$E_k = (v_F'^2 k^2 + \Delta_\sigma^2)^{1/2}. \quad (19)$$

Notice then that the energy eigenvalue for the new right-moving state  $c_k^\dagger |0\rangle$  is equal to  $\varepsilon_k = \theta(k)E_k - \theta(-k)E_k$ , while it is equal to  $-\varepsilon_k$  for the new left-moving state  $d_k^\dagger |0\rangle$ . Finally, the field operators obtained after such a canonical transformation are then  $\Psi_+(x) = L_x^{-1/2} \sum_k e^{ikx} c_k$  and  $\Psi_-(x) = L_x^{-1/2} \sum_k e^{ikx} d_k$ . After making the Luttinger liquid hypothesis, which assumes that only excitations near the Fermi surface are relevant, we obtain the following effective *massless* Thirring model Hamiltonian:

$$\begin{aligned} H'_\sigma = \int dx \Psi'^\dagger (\sigma_3 v_F'' i \partial_x + 2^{1/2} V_{KF}) \Psi' + 2g'_0 \int dx \Psi_-^\dagger \Psi_+^\dagger \Psi_+ \Psi_- \\ - \xi'_p \Delta_\sigma (\Psi_-^\dagger \Psi_+ + \Psi_+^\dagger \Psi_-)|_{x_0}, \end{aligned} \quad (20)$$

where  $v_F'' = (v'_F k_F / E_{k_F}) v'_F$ ,  $g'_0 = (v'_F k_F / E_{k_F})^2 g_0$  and  $\xi'_p = (v'_F k_F / E_{k_F}) \xi_p$  are the renormalized Fermi velocity, interaction and pinning scale, respectively, and where  $\Psi' =$

$(\Psi_-, \Psi_+)$  is the canonically transformed spinor field. The above effective Luttinger model is valid in the limit of (i) a thick pinning plane,  $\xi_p > a_0$ , with respect to the separation in between Josephson vortices, in the limit of (ii) high parallel fields  $B_{\parallel} > \Phi_0/\lambda_J d$ , and at (iii) temperatures in the vicinity of the depinning transition,  $T \sim T_{dp}$  [see Appendix B, Eqs. (B1) and (B14)]. Notice that the sheer existence of the gapless fermion analogy (20) indicates algebraic long-range order,  $\langle e^{i\theta_-(0)} e^{-i\theta_-(r)} \rangle \propto r^{-2K'_\sigma}$ , for the vortex lattice,<sup>20</sup> where  $K'_\sigma = [2(T/T_{dp}) - 1]^{1/2}$  at high fields. Finally, transforming *back* the massless Thirring model (20) to the bosonic description, one recovers the gaussian limit of the sine-Gordon model (12) in zero parallel field,

$$E_{LL} = J'_{\parallel} \int d^2r \frac{1}{4} (\vec{\nabla} \theta_-)^2 - \varepsilon_p^{\parallel} \int dy \cos \theta_-|_{x_0} + \frac{v_F'}{v_F''} \varepsilon_L^{\parallel} \int d^2r \theta (x - x_0) \frac{\partial \theta_-}{\partial x}, \quad (21)$$

but with a renormalized local stiffness  $J'_{\parallel}$  such that  $2\pi J'_{\parallel}/k_B T = 1 + g'_0/\pi v_F''$ . Comparison of the latter with Eq. (14) thus yields

$$\frac{2\pi J'_{\parallel}}{k_B T} = 1 + \frac{v_F''}{v_F'} \left( \frac{T_{dp}}{T} - 1 \right) \quad (22)$$

for one over the effective coupling constant of the gaussian theory (21), where

$$\frac{v_F''}{v_F'} = \left[ 1 + \left( \frac{B_{\parallel}^*}{B_{\parallel}} \right)^2 \right]^{-1/2} \quad (23)$$

relates the Fermi velocity to the parallel field.

Above, we have reduced the pinned sine-Gordon model (12) to the bosonic description of a Luttinger liquid with a single backscattering impurity.<sup>18</sup> In particular, the equivalence (3) yields the relationship  $Z_{SG} \propto Z_{LL}(U_{\perp})/Z_{LL}(0)$  between the respective partition functions. The linear density  $\lambda_{flux}^{-1} = \langle \partial_y \theta_- / 2\pi i \rangle_{x_0}$  for half-loop (or fluxon<sup>7</sup>) excitations of Josephson vortices along the pin<sup>28</sup> is thus given by the difference  $[\delta \ln Z_{LL} / \delta (ia_0)]|_0^{U_{\perp}}$ , where the pinning term is held fixed. Here, the field  $a_0(y)$  is defined by  $\partial_y a_0 = V_{KF}/v_F''$ , where  $V_{KF} = 2\pi v_F' \varepsilon_L^{\parallel} / k_B T$  gives the voltage drop in the fermion analogy (13). Kane and Fischer have computed such functional derivatives using perturbation theory,<sup>18</sup> where they obtain the result  $\delta \ln Z_{LL} / \delta (ia_0) \propto (V_{KF}/V_0)^{\mu'}$  with exponent  $\mu' = 4\pi J'_{\parallel} / k_B T - 1$ . Assuming that the voltage scale  $V_0$  in the fermion analogy is related to a (field dependent) current scale

$I'_0$  through the relationship  $V_0 = 2\pi v'_F \varepsilon_0 / k_B T$ , with  $\varepsilon_0 = c^{-1}(I'_0/L_y)(\Phi_0/2\pi)$ , the above results lead to the relationship

$$\lambda_{fluxn}^{-1} \propto \left(\frac{I}{I'_0}\right)^{\mu'} - \left(\frac{I}{I_0}\right)^{\mu} \quad (24)$$

for the density of fluxons along the pin, with

$$\mu = 2\frac{T_{dp}}{T} - 1 \quad (25)$$

and  $I_0$  corresponding to the exponent and to the current scale, respectively, in the limit of infinite parallel field and/or Josephson penetration length. The assumption of flux creep dynamics<sup>3</sup> then implies that the true voltage drop between consecutive layers is  $V = (h/2e)(\bar{c}/\lambda_{fluxn})$ , where  $\bar{c}$  is the average creep velocity. We therefore predict that the  $c$ -axis  $I$ - $V$  characteristic limited by parallel collective pinning is algebraic at relatively high parallel fields. In particular, Eq. (24) yields a voltage drop  $V \propto I^{\mu'}$  for low currents  $I \rightarrow 0$ . Also, both  $\mu' \rightarrow \mu$  and  $I'_0 \rightarrow I_0$  as  $B_{\parallel} \rightarrow \infty$ . Eq. (24) thus indicates that the system becomes more superconducting as parallel field increases. Finally, by Eq. (22) we have that  $\mu = 1 = \mu'$  at the depinning transition,  $T = T_{dp}$ . This means that ohmic flux flow sets in at temperatures above or equal to  $T_{dp}$  by Eq. (24). Given the temperature dependence of the parallel field scale  $B_{*}^{\parallel} \sim H_1^{\parallel}$  (see Fig. 1), the  $c$ -axis current should then peak at some temperature  $T_p < T_{dp}$  for fixed field and voltage.

## IV. Discussion

The previous results evoke the following image for the parallel thermodynamics of layered superconductors in the quasi-2D regime: Each Josephson vortex can be viewed as a string of width  $\Lambda_0$  confined to a given pair of consecutive layers. Parallel fluctuations of the string give rise to an effective Josephson penetration length,  $\lambda_J(T) > \Lambda_0$ , as well as to entropic pressure.<sup>14,15,24</sup> The latter is responsible for both the anomalous  $H_{\parallel}^{-1}$  tail shown by the parallel magnetization (11) and for the parallel collective pinning effect discussed above. Although HTSC films that are equivalent to  $N$  Josephson-coupled layers (1) already exist, their physical properties have been examined only at temperatures far

from the critical region.<sup>17,29</sup> Comparable studies should be carried out within the critical regime of these materials to test the predictions made here.

Finally, it must be stressed that all of the results obtained here are only valid for physics at length scales *large* in comparison to the ultraviolet cut-off  $\alpha$  of the fermion analogy; e.g., for parallel fields  $B_{\parallel} < \Phi_0/\alpha d$ . Clearly the in-plane coherence length,  $\xi_0$ , which is roughly equal to the size of a typical Cooper pair, provides a lower bound for  $\alpha$ . In addition, the string image mentioned above suggests that the bare Josephson penetration length  $\Lambda_0$  supplies an upper bound for  $\alpha$ . Where exactly within these limits  $\alpha$  lies remains to be determined.

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## Appendix A: Derivation of Ginzburg-Landau Equations for Coupled Charge-Density Waves in One Dimension

Below, we shall recover the LD free-energy functional (1) from the mean-field approximation (8) for the fermion analogy [Eqs. (2a) - (6)] by extending Gorkov's original derivation of the Ginzburg-Landau field equation<sup>25</sup> to the case of two coupled CDW states in one dimension. The limit of low parallel field,  $b_{\parallel} \ll \Lambda_0^{-1}$ , is assumed.

Consider two adjacent layers  $l = 1, 2$  in isolation, each of length  $L_y$  along the parallel field. Assuming periodic boundary conditions in this direction, we may then define the fictitious temperature  $T' = v_F'/L_y$  for the CDW state in the fermion analogy. Standard mean-field calculations then yield that the CDW is stable for fictitious temperatures below a critical temperature  $T'_c = (e\gamma/\pi)\Delta_0$ , where  $\gamma$  denotes Euler's constant. Note that the thermodynamic limit  $L_y \rightarrow \infty$  clearly corresponds to the fictitious temperature  $T' = 0$ . To begin the derivation, we first observe that the mean-field equation  $H_l\Psi_l = \varepsilon_l\Psi_l$  plus the definitions

$$G_l(x, t; x', t') = -\langle T\Psi_R(x, l, t)\Psi_R^\dagger(x', l, t') \rangle \quad (A1)$$

$$F_l(x, t; x', t') = -\langle T\Psi_R^\dagger(x, l, t)\Psi_L(x', l, t') \rangle \quad (A2)$$

for the normal and the anomalous Greens functions, respectively, ultimately lead to the Gorkov equations

$$(i\omega_n + iv_F\partial_x + \mu_l)G_l(x, x', i\omega_n) + \Delta_l(x)F_l^*(x, x', i\omega_n) = \delta(x - x'), \quad (A3)$$

$$(-i\omega_n - iv_F\partial_x + \mu_l)F_l^*(x, x', i\omega_n) - \Delta_l^*(x)G_l(x, x', i\omega_n) = 0, \quad (A4)$$

along with the associated gap equations

$$\Delta_1^*(x) = -U_{\parallel}F_1^*(x, t; x, t) - U_{\perp}F_2^*(x, t; x, t), \quad (A5)$$

$$\Delta_2^*(x) = -U_{\parallel}F_2^*(x, t; x, t) - U_{\perp}F_1^*(x, t; x, t). \quad (A6)$$

Here,  $t = iy/v_F'$  is the fictitious time variable, while  $i\omega_n$  denote the corresponding Matsubara frequencies. Applying Gorkov's method<sup>25</sup> to these set of equations yields the Ginzburg-Landau field equations

$$-(\partial_x - 2ik_{F,1})^2\Delta_1^* + v_F^{-2}\Delta_{\sigma}^2(\Delta_1^* + \Delta_2^*) = 0 \quad (A7)$$

$$-(\partial_x - 2ik_{F,2})^2\Delta_2^* + v_F^{-2}\Delta_{\sigma}^2(\Delta_1^* + \Delta_2^*) = 0 \quad (A8)$$

to lowest order in  $\Delta_l$ , where

$$\Delta_\sigma = \frac{4e^\gamma}{[7\zeta(3)]^{1/2}} \left( \frac{2\pi v_F U_\perp}{U_\parallel^2 - U_\perp^2} \right)^{1/2} \Delta_0 \quad (A9)$$

is the pseudo-spin gap. Above,  $\zeta(z)$  denotes the zeta function. Finally, these equations can be integrated, the result of which is the Ginzburg-Landau free-energy functional

$$F \propto \int dx \left\{ \frac{1}{2} \left( \frac{\partial \theta'_1}{\partial x} - 2k_{F,1} \right)^2 + \frac{1}{2} \left( \frac{\partial \theta'_2}{\partial x} - 2k_{F,2} \right)^2 + \lambda_J^{-2} [1 - \cos(\theta'_1 - \theta'_2)] \right\} \Delta_0^2, \quad (A10)$$

for gaps of the form  $\Delta_l(x) = (-1)^l \Delta_0 e^{i\theta'_l(x)}$ . Above,  $\lambda_J = v_F/|\Delta_\sigma|$  is the Josephson penetration length. After making the gauge transformation  $\theta'_l(x) = \theta_l(x) + 2k_{F,l}x$ , we recover the form (1) of the LD free-energy. Last, it is worth mentioning that  $\lambda_J$  and  $\Lambda_0$  are approximately equal along the LE line,  $T = 2\pi J_\parallel$ , in the limit  $|U_\perp| \ll |U_\parallel|$ . This is demonstrated by observing that we have  $U_\parallel = -6\pi v_F/5$  and  $\Lambda_0 = (\pi v'_F/|U_\perp|)^{1/2} \alpha$  in such case [see Eqs. (6)], and by substitution of the former into Eq. (A9). On the other hand, Eq. (A9) also indicates that  $\lambda_J$  diverges exponentially as  $T$  approaches the decoupling temperature  $T_*$  from below, since  $\Delta_0 \cong 2\omega_0 \exp[-2\pi v_F/(2U_\perp - U_\parallel)]$ . This agrees with results based on the Coulomb gas analogy<sup>14</sup> for the LD model (1), as well as with those based on a model for “frozen” layered superconductors in the Meissner phase.<sup>24</sup>

## Appendix B: Derivation of Effective Massless Thirring Model

Below, we shall obtain the effective low-energy Hamiltonian (20) of the massive Thirring model analogy (13) for a pinned double-layer superconductor (12) in the presence of parallel field. The limit of a thick pinning plane

$$\xi_p > a_0, \quad (B1)$$

with respect to the average separation in between Josephson vortices (fermions) is assumed to insure the validity of the Luttinger-Liquid hypothesis in the fermion analogy (13). Note that the above separation is related to the parallel magnetic induction and to the Fermi wavenumber by  $b_{\parallel} = 2\pi/a_0 = 2k_F$ . The gapless Luttinger model will be achieved through the canonical transformation [see Eqs. (15) - (19)] of the original left and right moving fields

$$\Psi_R(x) = L_x^{-1/2} \sum_k (x_k c_k - y_k d_k) e^{ikx} \quad (B2)$$

$$\Psi_L(x) = L_x^{-1/2} \sum_k (y_k c_k + x_k d_k) e^{ikx} \quad (B3)$$

into the new fields  $\Psi_+(x) = L_x^{-1/2} \sum_k c_k e^{ikx}$  and  $\Psi_-(x) = L_x^{-1/2} \sum_k d_k e^{ikx}$ . We now begin the derivation of Eq. (20) by applying this canonical transformation to each term in the fermion analogy (13).

*Kinetic Energy.* The kinetic energy is given by  $H_0 = \sum_k (\varepsilon_k^+ c_k^\dagger c_k + \varepsilon_k^- d_k^\dagger d_k)$ , where  $\varepsilon_k^\pm = \pm\theta(k)E_k \mp \theta(-k)E_k$  are the energy eigenvalues of the quasi-particle excitations. Since only those excitations that are near the Fermi surface are relevant, we can approximate the quasi-particle energy spectrum by  $\varepsilon_k^\pm \cong \varepsilon_F \pm v_F''(k \mp k_F)$ , where  $v_F'' = (v_F' k_F / E_{k_F}) v_F'$  is the group velocity at the Fermi surface. This immediately yields the new expression

$$H_0 = \int dx v_F'' (\Psi_-^\dagger i \partial_x \Psi_- - \Psi_+^\dagger i \partial_x \Psi_+) \quad (B4)$$

for the kinetic energy modulo a trivial shift of the chemical potential.

*Potential Energy.* Using the form  $V_{\text{KF}}(x) = \sum_q e^{iqx} V_{\text{KF}}(q)$  for potential energy drop at  $x = x_0$  in terms of its Fourier transform  $V_{\text{KF}}(q)$ , we can reexpress the corresponding term,  $H_{\text{KF}} = 2^{1/2} \int dx V_{\text{KF}} \Psi^\dagger \Psi$ , in the fermion analogy by

$$\begin{aligned} H_{\text{KF}} = 2^{1/2} \sum_{k,q} V_{\text{KF}}(q) [ & (x_{k+q} x_k + y_{k+q} y_k) c_{k+q}^\dagger c_k - (x_{k+q} y_k - y_{k+q} x_k) c_{k+q}^\dagger d_k \\ & - (y_{k+q} x_k - x_{k+q} y_k) d_{k+q}^\dagger c_k + (y_{k+q} y_k + x_{k+q} x_k) d_{k+q}^\dagger d_k ]. \end{aligned} \quad (B5)$$

After making the Luttinger-Liquid-type approximation  $(x_{k+q}, y_{k+q}) \rightarrow (x_k, y_k)$  above for the coherence factors, which is valid in the limit (B1)  $k_F \xi_p \gg 1$ , we recover the original simple form

$$H_{\text{KF}} \cong 2^{1/2} \int dx V_{\text{KF}} (\Psi_+^\dagger \Psi_+ + \Psi_-^\dagger \Psi_-) \quad (B6)$$

for the potential energy in terms of the new fields. Here, we have used the identity  $x_k^2 + y_k^2 = 1$ .

*Pinning/Backscattering.* Let us first reexpress the pinning energy (13) as

$$H_{\text{pin}} = -\Delta_\sigma \int_{x_0 - \xi_p/2}^{x_0 + \xi_p/2} dx (\Psi_L^\dagger \Psi_R + \Psi_R^\dagger \Psi_L). \quad (B7)$$

If we now take the long-wavelength limit (B1)  $k_F \xi_p \gg 1$ , then the bounds on the above integral can be extended to  $\pm\infty$ . After Fourier transformation, we obtain the expression

$$H_{\text{pin}} \rightarrow -\Delta_\sigma \sum_k [(x_k^2 - y_k^2)(d_k^\dagger c_k + c_k^\dagger d_k) + 2x_k y_k (c_k^\dagger c_k - d_k^\dagger d_k)] \quad (B8)$$

as a result. If we then make the additional replacements

$$x_k \rightarrow u_{k_F} \quad \text{and} \quad y_k \rightarrow (\text{sgn } k) v_{k_F} \quad (B9)$$

valid under the Luttinger-Liquid hypothesis, tracing back the previous steps leads to the effective low-energy Hamiltonian

$$H_{\text{pin}} \cong -(u_{k_F}^2 - v_{k_F}^2) \xi_p \Delta_\sigma (\Psi_-^\dagger \Psi_+ + \Psi_+^\dagger \Psi_-)|_{x_0} - 2u_{k_F} v_{k_F} \xi_p \Delta_\sigma (\Psi_+^\dagger \Psi_+ + \Psi_-^\dagger \Psi_-)|_{x_0} \quad (B10)$$

corresponding to the pinning term. Last, the second term above represents a trivial shift of the chemical potential in the long wave-length limit (B1). The effective pinning term at the Fermi surface thus has the same form as the original, but with a renormalized pinning scale  $\xi'_p = (v'_F k_F / E_{k_F}) \xi_p$ .

*Interaction.* Substituting the canonical transformation (B2) and (B3) into the forward scattering interaction term  $H_1 = 2g_0 \int dx \Psi_L^\dagger \Psi_R^\dagger \Psi_R \Psi_L$  yields an equivalent expression of the form

$$\begin{aligned} H_1 = \frac{2g_0}{L_x} \sum_{k, k', q} & (y_{k+q} y_k c_{k+q}^\dagger c_k + y_{k+q} x_k c_{k+q}^\dagger d_k + x_{k+q} y_k d_{k+q}^\dagger c_k + x_{k+q} x_k d_{k+q}^\dagger d_k) \times \\ & \times (x_{k'-q} x_{k'} c_{k'-q}^\dagger c_{k'} - x_{k'-q} y_{k'} c_{k'-q}^\dagger d_{k'} - y_{k'-q} x_{k'} d_{k'-q}^\dagger c_{k'} + y_{k'-q} y_{k'} d_{k'-q}^\dagger d_{k'}). \end{aligned} \quad (B11)$$

After making the replacements (B9) in the coherence factors above, which is valid for excitations near the Fermi surface, we recover the original form

$$H_1 \cong 2g_0 \left( \frac{v'_F k_F}{E_{k_F}} \right)^2 \int dx \Psi_-^\dagger \Psi_+^\dagger \Psi_+ \Psi_- + H_4 \quad (B12)$$

for the interaction energy in terms of the new fields, in addition to a forward scattering contribution

$$H_4 = 2g_0 (u_{k_F} v_{k_F})^2 \sum_q [\rho_+(q) \rho_+(-q) + \rho_-(q) \rho_-(-q)] \quad (B13)$$

that appears in terms of the new particle-hole operators  $\rho_+(q) = L_x^{-1/2} \sum_k c_{k+q}^\dagger c_k$  and  $\rho_-(q) = L_x^{-1/2} \sum_k d_{k+q}^\dagger d_k$  for right and left moving fermions. This interaction can be incorporated into the kinetic energy (B4) via Kronig's identity,<sup>20</sup> which yields the final result

$$v_F'' = v_F' \left( \frac{v_F' k_F}{E_{k_F}} + \frac{g_0}{2\pi v_F'} \frac{\Delta_\sigma^2}{E_{k_F}^2} \right) \quad (B14)$$

for the effective Fermi velocity. Notice, however, that this correction is negligible in the limit of high parallel fields,  $\Delta_\sigma \ll E_{k_F}$ , and at temperatures near the depinning transition,  $|g_0| \ll \pi v_F'$ .

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## Figure Caption

- Fig. 1. Shown is the phase diagram for thin-film layered superconductors in the quasi-2D regime as a function of the parallel field,  $H_{\parallel}$ . The integer  $l_1$  designates a parallel vortex lattice in which flux penetrates every  $l_1$  layers (see ref. 16). Also,  $T_{c0}$  denotes the mean-field transition temperature of an isolated layer.
- Fig. 2. The equilibrium parallel magnetization [Eq. (11)] obtained from the mean-field theory approximation to the fermion analogy is displayed in the vicinity of the decoupling transition,  $T \lesssim T_*$  and/or  $B_{\perp} \gtrsim \Phi_0/\lambda_J^2$ .



